

ON CYCLIC UNIVERSES

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Dark energy in a brane world reconciles an infinitely cyclic cosmology with the second law of thermodynamics. At turnaround one causal patch with no matter and vanishing entropy is retained for the contraction.

1. Introduction

An old question in theoretical cosmology is whether an infinitely oscillatory universe which avoids an initial singularity can be consistently constructed. As realized by Friedmann¹ and especially by Tolman^{2,3} one principal obstacle is the second law of thermodynamics which dictates that the entropy increases from cycle to cycle. If the cycles thereby become longer, extrapolation into the past will lead back to an initial singularity again, thus removing the motivation to consider an oscillatory universe in the first place. This has led to the abandonment of the oscillatory universe by many workers.

Some work has been started to exploit the dark energy in allowing cyclicity possibly without the need for inflation in ^{4,5,6,7,8}. Another new ingredient is the use of branes and a fourth spatial dimension as in ^{9,10,11,12} which examined consequences for cosmology. The Big Rip and replacement of dark energy by modified gravity were explored in ^{13,14}.

If the dark energy has a super-negative equation of state, $\omega_\Lambda = p_\Lambda/\rho_\Lambda < -1$, it leads to a Big Rip¹⁵ at a finite future time where there exist extraordinary conditions with regard to density and causality as one approaches the Rip. In the present article we explore whether these exceptional conditions can assist in providing an infinitely cyclic model. We consider a model¹⁶ where, as we approach the Rip, expansion stops due to a brane contribution just short of the Big Rip and there is a turnaround at time $t = t_T$ when the scale factor is deflated to a very tiny fraction (f) of itself

and only one causal patch is retained, while the other $1/f^3$ patches contract independently to separate universes. Turnaround takes place an extremely short time ($< 10^{-27}s$) before the Big Rip would have occurred, at a time when the universe is fractionated into many independent causal patches¹⁴. We discuss contraction which occurs with a very much smaller universe than in expansion and with almost vanishing entropy because it is assumed empty of dust, matter and black holes.

A bounce takes place a short time before a would-be Big Bang. After the bounce, entropy is injected by inflation¹⁷, where is assumed that an inflaton field is excited. Inflation is thus be a part of the present model which is one distinction from the work of^{5,6,7,8}. For cyclicity of the entropy, $S(t) = S(t + \tau)$ to be consistent with thermodynamics it is necessary that the deflationary decrease by f^3 compensate the entropy increase acquired during expansion including the increase during inflation.

A possible shortcoming of the proposal could have been the persistence of spacetime singularities in cyclic cosmologies¹⁸, but to our understanding for the model we outline this problem is avoided, provided that the time average of the Hubble parameter during expansion is equal in magnitude and opposite in sign to its average during contraction.

This model gives renewed hope for the infinitely oscillatory universe sought in^{1,2,3}.

2. Expansion

Let the period of the Universe be designated by τ and the bounce take place at $t = 0$ and turnaround at $t = t_T$. Thus the expansion phase is for times $0 < t < t_T$ and the contraction phase corresponds to times $t_T < t < \tau$. We employ the following Friedmann equation for the *expansion* period $0 < t < t_T$:

$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi G}{3} \left[\left(\frac{(\rho_\Lambda)_0}{a(t)^{3(\omega_\Lambda+1)}} + \frac{(\rho_m)_0}{a(t)^3} + \frac{(\rho_r)_0}{a(t)^4} \right) - \frac{\rho_{total}(t)^2}{\rho_c} \right] \quad (1)$$

where the scale factor is normalized to $a(t_0) = 1$ at the present time $t = t_0 \simeq 14Gy$. To explain the notation, $(\rho_i)_0$ denotes the value of the density ρ_i at time $t = t_0$. The first two terms are the dark energy and total matter (dark plus luminous) satisfying

$$\Omega_\Lambda = \frac{8\pi G(\rho_\Lambda)_0}{3H_0^2} = 0.72 \quad \text{and} \quad \Omega_m = \frac{8\pi G(\rho_m)_0}{3H_0^2} = 0.28 \quad (2)$$

where $H_0 = \dot{a}(t_0)/a(t_0)$. The third term in the Friedmann equation is the radiation density which is now $\Omega_r = 1.3 \times 10^{-4}$. The final term $\sim \rho_{total}(t)^2$

is derivable from a brane set-up^{9,10,12}; we use a negative sign arising from negative brane tension (a negative sign can arise also from a second timelike dimension but that gives difficulties with closed timelike paths). $\rho_{total} = \Sigma_{i=\Lambda, m, r} \rho_i$. As the turnaround is approached, the only significant terms in Eq.(1) are the first (where $\omega_\Lambda < -1$) and the last. As the bounce is approached, the only important terms in Eq.(1) are the third and the last. (We shall later argue that the second term, for matter, is absent during contraction.) In particular, the final term of Eq. (1), $\sim \rho_{total}(t)^2$, arising from the brane set-up is insignificant for almost the entire cycle but becomes dominant as one approaches $t \rightarrow t_T$ for the turnaround and again for $t \rightarrow \tau$ approaching the bounce.

3. Turnaround

Let us assume for algebraic simplicity $\omega_\Lambda = -4/3 = \text{constant}$. This value is already almost excluded by WMAP3¹⁹ but to begin we are aiming only at consistency of infinite cyclicity. More realistic values may be discussed elsewhere. With the value $\omega_\Lambda = -4/3$ we learn from¹³ that the time to the Big Rip is $(t_{rip} - t_0) = 11\text{Gy}(-\omega_\Lambda - 1)^{-1} = 33\text{Gy}$ which is, as we shall discuss, within 10^{-27} second or less, when turnaround occurs at $t = t_T$. So if we adopt $t_0 = 14\text{Gy}$ then $t_T = t_0 + (t_{rip} - t_0) \sim (14 + 33)\text{Gy} = 47\text{Gy}$. From the analysis in^{13,14,15} the time when a system becomes gravitationally unbound corresponds approximately to the time when the dark energy density matches the mean density of the bound system. For an object like the Earth or a hydrogen atom water density ρ_{H_2O} is a practical unit.

With this in mind, for the simple case of $\omega = -4/3$ we see from Eq.(1) that the dark energy density grows proportional to the scale factor $\rho_\Lambda(t) \propto a(t)$ and so given that the dark energy at present is $\rho_\Lambda \sim 10^{-29}\text{g/cm}^3$ it follows that $\rho_\Lambda(t_{H_2O}) = \rho_{H_2O}$ when $a(t_{H_2O}) \sim 10^{29}$. We can estimate the time t_{H_2O} by taking on the RHS of the Friedmann equation only dark energy $\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \Omega_\Lambda a^{-\beta}$ with $\beta = 3(1 + \omega)$. When we specialize to $\omega = -4/3$ it follows that

$$\frac{a(t_{H_2O})}{(a(t_0) = 1)} = \left(\frac{(t_{rip} - t_0)}{(t_{rip} - t_{H_2O})} \right)^2 \quad (3)$$

so that $(t_{rip} - t_{H_2O}) = 33\text{Gy} \times 10^{-14.5} \simeq 10^{3.5}\text{s} \sim 1 \text{ hour}$. [The value is sensitive to ω] It is instructive to consider approach to the Rip a more general critical density $\rho_c = \eta \rho_{H_2O}$ and to compute the time $(t_{rip} - t_\eta)$ such

that $\rho_\Lambda(t_\eta) = \rho_c = \eta \rho_{H_2O}$. We then find, using $a(t_\eta) = 10^{29}\eta$, that

$$(t_{rip} - t_\eta) = (t_{rip} - t_0)10^{-14.5}\eta^{-1} \simeq \eta^{-1}\text{hours} \quad (4)$$

which is the required result. We shall see $\eta > 10^{31}$ so the time in (4) is $< 10^{-27}s$.

To discuss the turnaround analytically we keep only the first and last terms, the only significant ones, on the RHS of Eq.(1) which becomes for the special case $\omega = 4/3$

$$\left(\frac{\dot{a}}{a}\right)^2 = \alpha_1 a - \alpha_2 a^2 \quad (5)$$

in which

$$\alpha_1 = \frac{8\pi G}{3}(\rho_\Lambda)_0 \quad \alpha_2 = \frac{8\pi G}{3} \frac{(\rho_\Lambda)_0^2}{\rho_c} \quad (6)$$

Writing $a = z^2$ and $z = (\alpha_1/\alpha_2)^{1/2} \sin\theta$ gives

$$dt = \frac{2\sqrt{\alpha_2}}{\alpha_1} \frac{d\theta}{\sin^2\theta} = \frac{2\sqrt{\alpha_2}}{\alpha_1} d(-\cot\theta) \quad (7)$$

Integration then gives for the scale factor

$$a(t) = \left(\frac{\alpha_1}{\alpha_2}\right) \sin^2\theta = \frac{\rho_c}{(\rho_\Lambda)_0} \left[\frac{1}{1 + \left(\frac{t_T - t}{C}\right)^2} \right] \quad (8)$$

where $C = -(3/2\pi G\rho_c)^{1/2}$. At turnaround $t = t_T$, $a(t_T) = [\rho_C/(\rho_\Lambda)_0] = (a(t))_{max}$. At the present time $t = t_0$, $a(t_0) = 1$ and $\sin^2\theta_0 = [(\rho_\Lambda)_0/\rho_C] \ll 1$, increasing during subsequent expansion to $\theta_T = \pi/4$.

A key ingredient in our model is that at turnaround $t = t_T$ our universe deflates dramatically with effective scale factor $a(t_T)$ shrinking before contraction to $\hat{a}(t_T) = f a(t_T)$ where $f < 10^{-28}$. This jettisoning of almost all, a fraction $(1 - f)$, of the accumulated entropy is permitted by the exceptional causal structure of the universe. We shall see later that the parameter η at turnaround lies in the range $\eta = 10^{31}$ to $\eta = 10^{87}$ which implies a dark energy density at turnaround (Planckian density of $\rho_\Lambda \sim 10^{104}\rho_{H_2O}$ can be avoided) such that, according to the Big Rip analysis of ^{13,14}, all known, and yet unknown smaller, bound systems have become unbound and the constituents causally disconnected. Recall that the density of a hydrogen atom is approximately ρ_{H_2O} and we are reaching a dark energy density of from 31 to 87 orders of magnitude higher.

According to these estimates, at $t = t_T$ the universe has already fragmented into an astronomical number $(1/f^3)$ of causal patches, each of which

independently contracts as a separate universe leading to an infinite multiverse. The entropy at $t = t_T$ is thus divided between these new contracting universes and our universe retains only a fraction f^3 . Since our model universe has cycled an infinite number of times, the number of parallel universes is infinite.

4. Contraction and Bounce

The contraction phase for our universe occurs for the period $t_T < t < \tau$. The scale factor for the contraction phase will be denoted by $\hat{a}(t)$ while we use always the same linear time t subject to the periodicity $t + \tau \equiv t$. At the turnaround we retain a fraction f^3 of the entropy with $\hat{a}(t_T) = fa(t_T)$ and for the contraction phase the Friedmann equation is

$$\left(\frac{\dot{\hat{a}}(t)}{\hat{a}(t)}\right)^2 = \frac{8\pi G}{3} \left[\left(\frac{(\hat{\rho}_\Lambda)_0}{\hat{a}(t)^{3(\omega_\Lambda+1)}} + \frac{(\hat{\rho}_r)_0}{\hat{a}(t)^4} \right) - \frac{\hat{\rho}_{total}(t)^2}{\hat{\rho}_c} \right] \quad (9)$$

where we have defined

$$\hat{\rho}_i(t) = \frac{(\rho_i)_0 f^{3(\omega_i+1)}}{\hat{a}(t)^{3(\omega_i+1)}} = \frac{(\hat{\rho}_i)_0}{\hat{a}(t)^{3(\omega_i+1)}} \quad (10)$$

but in contrast to Eq.(1) we have set $\hat{\rho}_m = 0$ because our hypothesis is that the causal patch retained in the model contains only dark energy and radiation but no matter including no black holes. This is necessary because during a contracting phase dust or matter would clump, even more readily than during expansion, and inevitably interfere with cyclicity. Perhaps more importantly, presence of dust or matter would require that our universe go in reverse through several phase transitions (recombination, QCD and electroweak to name a few) which would violate the second law of thermodynamics. We thus require that *our universe comes back empty!* Any tiny entropy associated with radiation is constant during adiabatic contraction.

The contraction of our universe will proceed from one of the $1/f^3$ causal patches following Eq.(9) until the radiation balances the brane tension at the bounce.

At the bounce, the contraction scale is given, using $\rho_c = \eta\rho_{H_2O}$, from Eq. (1) as

$$a(\tau)^4 = \left(\frac{(\rho_r)_0}{\eta\rho_{H_2O}} \right) \quad (11)$$

Now the model's bounce at $t = \tau$ must be before the electroweak transition at $t_{EW} = 10^{-10}s$ when $a(t_{EW}) = 10^{-15}$, and after the Planck scale

when $a(t_{Planck}) = 10^{-32}$ in order to accommodate the well established weak transition and to avoid uncertainties associated with quantum gravity. With this in mind, here are three illustrative values for the bounce temperature T_B : at a GUT scale $T_B = 10^{17} GeV, a(t_B) = 10^{-30}$, at an intermediate scale $T_B = 10^{10} GeV, a(t_B) = 10^{-23}$, and at a weak scale $T_B = 10^3 GeV, a(t_B) = 10^{-16}$. From Eq.(11) and Eq.(4) for these three cases one finds, respectively, $\eta = 10^{87}$ and $(t_{rip} - t_T) = 10^{-87} hr$, $\eta = 10^{59}$ and $(t_{rip} - t_T) = 10^{-59} hr$, and $\eta = 10^{31}$ and $(t_{rip} - t_T) = 10^{-31} hr$.

Immediately after the bounce, we assume that an inflaton field is excited and there is conventional inflation with enhancement $E = a(\tau + \delta)/\hat{a}(\tau)$. Successful inflation requires $E > 10^{28}$. Consistency requires therefore $f < E^{-1}$ to allow for the entropy accrued during expansion after inflation. The fraction of entropy jettisoned from our universe at deflation is thus extremely close to one, being less than one and more than $(1 - 10^{-28})^3$.

5. Entropy

The present entropy associated with radiation $S \sim 10^{88}$ is much less than the holographic bound $S \sim 10^{123}$. In ²⁰ it is suggested that at least some of this difference may come from supermassive black holes. The entropy contribution from the baryons is smaller than S_γ by some ten orders of magnitude, so like that of the dark matter, is negligible.

What is the entropy of the dark energy? If it is perfectly homogeneous and non-interacting it has zero temperature and entropy. Finally, the 4th term in Eq.(1) corresponding to the brane term is negligible, as we have already estimated. The conclusion is that at present $S_{total}(t_0) \sim 10^{88}$.

Our main point is that in order for entropy to be cyclic, the entropy which was enhanced by a huge factor $E^3 > 10^{84}$ at inflation must be reduced dramatically at some point during the cycle so that $S(t) = S(t + \tau)$ becomes possible. Since it increases during expansion and contraction, the only logical possibility is the decrease at turnaround as accomplished by our causal patch idea. The second law of thermodynamics continues to obtain for other causal patches, each with practically vanishing entropy at turnaround, but these are permanently removed from our universe contracting instead into separate universes.

For contraction $t_T < t < \tau$ we are assuming the universe during contraction is empty of matter until the bounce so its entropy is vanishingly small. Immediately after the bounce inflation arises from an inflaton field, assumed to be excited. We find the counterpoise of inflation at the bounce and deflation at turnaround an appealing aspect of the present model.

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